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NSEA 2026 DETAILED SOLUTION

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INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN ASTRONOMY (NSEA) 2026

(QUESTION PAPER CODE 43)

Date : 21/11/2026 **Time : 120 Minute** **Maximum Marks: 216**

Write the question paper code (mentioned above) on YOUR OMR Answer Sheet (in the space provided), otherwise your Answer Sheet will NOT be evaluated. Note that the same Question paper code appears on each page of the question paper.

INSTRUCTIONS

1. Use of mobile phone, smart watches, and iPad during examination is **STRICTLY PROHIBITED.**
2. In addition to this question paper, you are given OMR Answer Sheet along with candidate's copy.
3. On the OMR sheet, make all the entries carefully in the space provided ONLY in BLOCK CAPITALS as well as by properly darkening the appropriate bubbles.
   Incomplete/ incorrect/ carelessly filled information may disqualify your candidature.
4. On the OMR Answer sheet, use only **BLUE or BLACK BALL POINT PEN** for making entries and filling bubbles.
5. Your **Ten-digit roll number and date of birth** entered in the OMR Answer sheet shall remain your login credentials (means login id and password respectively) for accessing your performance/result in National Standard Examination in Astronomy – 2025.
6. Question paper has two parts. In part A-1 (**Q. No.1 to 48**) each question has four alternatives, out of which only one is correct. Choose the correct alternative (s) and fill the appropriate bubbles(s), as shown.

   Q.No.22 

   In part A-2 (**Q. No. 49 to 60**) each question has four alternative out of which any number of alternative (s) (1, 2, 3, or 4) may be correct. You have to choose all correct alternative(s) and fill the appropriate bubbles(s), as shown

   Q.No.54 

7. For **Part A-1,** each correct answer carries 3 marks whereas 1 mark will be deducted for each wrong answer. In **Part A-2,** you get 6 marks. If all the correct alternative are marked. No negative marks in this part.
8. Rough work should be done only in the space provided. There are **30** printed pages in this paper.
9. Use of **non-programmable scientific** calculator is allowed
10. No candidate should leave the examination hall before the completion of the examination.
11. After submitting answer paper, take away the question paper & candidate's copy of OMR for your reference

Please DO NOT make any mark other than filling the appropriate bubbles properly in the space provided on the OMR answer sheet.

OMR answer sheets are evaluated using machine, hence CHANGE OF ENTRY IS NOT ALLOWED, Scratching or overwriting may result in wrong score.

DO NOT WRITE ON THE BACK SIDE OF THE OMR ANSWER SHEET.

Name of Student :.....

Batch : .....

Enrolment No. | | | | | | | | | | | | | | | | |

INDIAN ASSOCIATION OF PHYSICS TEACHERS

NATIONAL STANDARD EXAMINATION IN ASTRONOMY (NSEA) 2026

PAPER CODE - 43

Date of Examination – 21th November, 2026

SOLUTIONS

Attempt All Sixty Questions

(NSEA) PART : A–1

ONLY ONE OUT OF FOUR OPTIONS IS CORRECT. BUBLE THE CORRECT OPTION.

[Q.1] A compressor designed to compress air is used to compress Helium instead of air near room temperature. Select correct statement regarding the functioning of this compressor with Helium. When the compressor is used for the same gauge reading, it will

[A] be overheated [B] develop less pressure than that for air

[C] be cooled [D] compress the gas to smaller volume

[ANS] **A**

[SOLN] Adiabatic compression

$$P^{1-\gamma}T^{\gamma} = \text{const} \Rightarrow P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma}$$

$$\Rightarrow T_2 = T_1\left[\frac{P_1}{P_2}\right]^{\frac{1-\gamma}{\gamma}}$$

$\gamma_{air} \approx 1.4s$ & $\gamma_{1+e} \approx 1.67$

$$\text{So, } T_2(\text{air}) = T_1\left[\frac{P_2}{P_1}\right]^{0.286} \quad \& \; T_2^1(\text{helium}) = T_1\left[\frac{P_2}{P_1}\right]^{0.401}$$

Since $P_2 > P_1$

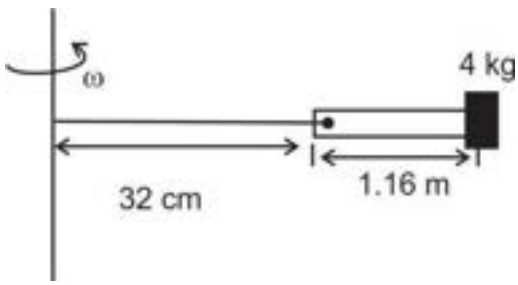
So, if will be overheated

[Q.2] An athlete weighing 60 kg has participated in hammer throw. Weight of the hammer is 4.0 kg. It is connected to a non-stretchable wire. The distance between grip and the hammer head is 1.16 m. At the time of releasing the hammer, athlete stretches her hands such that the grip of the hammer is at 32 cm from the axis of rotation. To achieve the linear velocity of 27 m/s, the centrifugal force on the hammer and the time it should complete each rotation is,

[A] 36496 N, 0.36 s [B] 30512 N, 0.27 s [C] 1970 N, 0.34 s [D] 1906 N, 0.27 s

[ANS] **C**

[SOLN]



$$v = \omega r \Rightarrow 27\,m/s = \omega \times (1.16 + 0.32)$$

$$\omega = \frac{27}{1.48} = 18.243 \qquad \text{So, } T = \frac{2\omega}{\omega} = 0.34\,\text{sec}$$

$$\text{And centrifugal force } = m\omega^2 r = 4 \times (18.243)^2 \times 1.48 = 1970N$$

[Q.3] An astronomical telescope with a primary mirror of 600 mm focal length is focussed for distant object using an eyepiece of focal length 15 mm. When a concave lens of 30 mm focal length is introduced between the primary and eyepiece to triple magnification of the system by what distance eyepiece needs to be shifted when focussed for distant object?

[A] 15 mm [B] 30 mm [C] 40 mm [D] 45 mm

[ANS] **C**

[SOLN] Initial telescope

$$M = \frac{f_0}{f_e}(\text{primary mirror}) = \frac{600}{15} = 40$$

Required from new telescope ;

$$M = \frac{F_0}{f_e}$$

$$\Rightarrow 120 = \frac{F_0}{f_e} \Rightarrow F_0 = 120 \times 15 = 1800\,mm$$

F_0 : Effective focal length of primary mirror.

$$F_0 = \frac{f_1 f_2}{f_1 + f_2 - d} = \frac{300 \times (-30)}{600 - 30 - d} \Rightarrow d = 580\,mm$$

For concave intermediate lens: (u = 20 mm)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{20} = \frac{1}{-30} \Rightarrow v = 60\,mm$$

Image distance from primary mirror = 580+ 60 = 640 mm

$\therefore$ Adjustment needed = 640 – 600 =40 mm

[Q.4] If $\alpha,\beta,\gamma \in \left(0,\frac{\pi}{2}\right)$, such that $\tan\alpha + \tan\beta + \tan\gamma = \tan\alpha \tan\beta \tan\gamma$, then the value of $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma$ is

[A] 0 [B] 1 [C] 2 [D] –1

[ANS] **B**

[SOLN] The identity tan α + tan β + tan γ = tan α tan β tan γ holds when $\alpha + \beta + \gamma = n\pi$.

Since $\alpha,\beta,\gamma \in (0,\pi/2)$, we have $0 < \alpha + \beta + \gamma < \frac{3\pi}{2}$, so $\alpha + \beta + \gamma = \pi$.

In any triangle $(\alpha + \beta + \gamma = \pi)$:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 - 2\cos\alpha\cos\beta\cos\gamma.$$

Adding $2\cos\alpha\cos\beta\cos\gamma$ to both sides gives:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma = 1$$

[Q.5] If $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = 0$, and $xyz \neq 0$; then

[A] $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1$ [B] $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$

[C] $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=-1$ [D] $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=-2$

[ANS] **C**

[SOLN] Factor x, y, z out of R_1, R_2, R_3 respectively:

Applying $R_1 \rightarrow R_1 + R_2 + R_3$ gives a common factor of (1 + 1/x + 1/y + 1/z) in the first row.
Evaluating the remaining determinant yields:
xyz (1 + 1/x + 1/y + 1/z) = 0.

[Q.6] $\sum_{k=1}^{n} \frac{1}{k(k+1)(k+2)} =$

[A] $\frac{1}{4}+\frac{1}{2(n+1)(n+2)}$ [B] $\frac{1}{4}-\frac{1}{2(n+1)(n+2)}$

[C] $\frac{1}{2}-\frac{1}{(n+1)(n+2)}$ [D] $\frac{1}{2(n+1)}$

[ANS] **A**

[SOLN] Using partial fractions / telescoping decomposition:

$T_r = 1 / [r(r + 1)(r + 2)] = (1/2) [1 / (r(r + 1)) − 1 / ((r + 1)(r + 2))]$.

Summing from r = 1 to n, intermediate terms cancel:

$S_n = (1/2) [1/(1 \cdot 2) − 1/((n + 1)(n + 2))] = 1/4 − 1 / [2(n + 1)(n + 2)]$.

[Q.7] The area (in appropriate units) enclosed by $y = x^3 - 4x + 3$ and $y = 3 - x$ is

[A] 3/2 [B] 9/2 [C] 6 [D] 9

[ANS] **B**

[SOLN] Find points of intersection: $x^2 - 4x + 3 = 3 - x \Rightarrow x^2 - 3x = 0 \Rightarrow x = 0, 3.$

On [0, 3], the line y = 3 − x lies above the parabola $y = x^2 - 4x + 3$.

Area = $\int_0^3 \left[(3 - x) - \left(x^2 - 4x + 3 \right) \right] dx = \int_0^3 \left(3x - x^2 \right) dx$

$\left[\frac{3}{2}x^2 - \frac{x^3}{3} \right]_0^3 = \frac{27}{2} - 9 = 9/2.$

[Q.8] If $a_n = \frac{1}{\sqrt{n}+\sqrt{n+1}}$, then $\sum_{n=1}^{99} a_n = ?$

[A] 9 [B] 10 [C] $\sqrt{99}-1$ [D] 11

[ANS] **A**

[SOLN] Rationalizing the denominator: $a_n = \left(\sqrt{(n+1)}-\sqrt{n}\right)/\left((n+1)-n\right) = \sqrt{(n+1)}-\sqrt{n}$

Summing from n = 1 to 99:

$S_{99} = \left(\sqrt{2}-\sqrt{1}\right)+\left(\sqrt{3}-\sqrt{2}\right)+.....+\left(\sqrt{100}-\sqrt{99}\right)$

$= \sqrt{100}-1 = 10-1 = 9$.

[Q.9] If $f(x) = x^x$ then $f'(1) = ?$

[A] 0 [B] 1 [C] 2 [D] e

[ANS] **B**

[SOLN] Taking natural logarithm on both sides: ln f(x) = x ln x.

Differentiating with respect to x:

$f'(x)/f(x) = 1 \cdot \ln x + x \cdot (1/x) = \ln x + 1 \Rightarrow f'(x) = x^x(1+\ln x)$.

At x = 1: f′(1) = $1^1(1 + \ln 1) = 1(1 + 0) = 1$.

[Q.10] The number of 5 digit numbers formed using 1,2,3,4,5 without repetition and divisible by 6 is

[A] 24 [B] 36 [C] 40 [D] 48

[ANS] **D**

[SOLN] Sum of the digits = 1 + 2 + 3 + 4 + 5 = 15, which is always divisible by 3. For a 5-digit number formed from {1, 2, 3, 4, 5} to be divisible by 6, it must also be even, so its last digit must be 2 or 4 (2 choices). The remaining 4 positions can be filled in $4! = 24$ ways.

Total numbers = 2 × 24 = 48.

[Q.11] A number is chose randomly from 1, 2, 3,….., 100. The probability that it is divisible by neither 2 nor 5 is

[A] $\frac{2}{5}$ [B] $\frac{1}{2}$ [C] $\frac{3}{5}$ [D] $\frac{5}{2}$

[ANS] **A**

[SOLN] In {1, 2, ……., 100}:

Number divisible by 2: $\left\lfloor \frac{100}{2} \right\rfloor = 50$.

Number divisible by 5: $\left\lfloor \frac{100}{5} \right\rfloor = 20$.

Numbers divisible by both 2 and 5 (i.e., 10): ⌊100/10⌋ = 10.

Numbers divisible by 2 or 5 = 50 + 20 − 10 = 60.

Numbers divisible by neither 2 nor 5 = 100 − 60 = 40.

Probability = 40 / 100 = 2/5.

[Q.12] If $z+\frac{1}{z}=2$, $z\neq 0$ and $z \in$ C, then $z^{2026}+\frac{1}{z^{2026}}=$

[A] 0 [B] 1 [C] 2 [D] 2026

[ANS] **C**

[SOLN] Multiply z + 1/z = 2 by z:

$z^2 - 2z + 1 = 0 \Rightarrow (z-1)^2 = 0 \Rightarrow z = 1$.

Substituting z = 1 gives $1^{2026} + 1/1^{2026} = 1 + 1 = 2$.

[Q.13] The number of solutions of sin x + sin 2x + sin 3x = 0 in $[0, 2\pi)$ is

[A] 4 [B] 5 [C] 6 [D] 7

[ANS] **C**

[SOLN] Group sin x + sin 3x = 2 sin 2x cos x:

2 sin 2x cos x + sin 2x = 0 $\Rightarrow$ sin 2x (2 cos x + 1) = 0.

Case 1: sin 2x = 0 $\Rightarrow$ 2x = 0, π, 2π, 3π, $4\pi \Rightarrow$ $x \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$ (4 solutions).

Case 2: cos x = −1/2 ⇒ x ∈ {2π/3, 4π/3} (2 solutions).

All 7 values are distinct in [0, 2π). Total solutions = 6.

[Q.14] If an integer n leaves reminder 1, when divided by 2 and 3, then $n^2 - 1$ is always divisible by

[A] 6 [B] 13 [C] 24 [D] 36

[ANS] **C**

[SOLN] Since n ≡ 1 (mod 2) and n ≡ 1 (mod 3), n = 6k + 1 for some integer k.

Then $n^2 - 1 = (6k + 1)^2 - 1 = 36k^2 + 12k = 12k(3k + 1)$.

Since either k or (3k + 1) is always even, k(3k + 1) is always divisible by 2.

Thus, 12k(3k + 1) is always divisible by 12 × 2 = 24.

(Note: for k = 1, n = 7 ⇒ $n^2 - 1 = 48$, which is divisible by 24, not 36).

[Q.15] Consider f(x) = $\sin^{-1}(\sin x) + \cos^{-1}(\cos x)$ for $x \in [-\pi, 2\pi]$. Which of the following correctly describes f ?

[A] f(x) = 2x for all $x \in [-\pi, 2\pi]$

[B] f is piecewise linear and takes the value π on the interval $\left[\frac{\pi}{2}, \pi\right]$

[C] f(x) = 0 for all $x \in [0, \pi]$

[D] The range of f on $[-\pi, 2\pi]$ is $[0, 2\pi]$

[ANS] **B**

[SOLN] Recall principal value branches: $\sin^{-1}(\sin x) \in [-\pi/2, \pi/2]$ and $\cos^{-1}(\cos x) \in [0, \pi]$

• For $x \in [0, \pi/2]$: $\sin^{-1}(\sin x) = x$ and $\cos^{-1}(\cos x) = x \Rightarrow f(x) = 2x$.

• For $x \in [\pi/2, \pi]$: $\sin^{-1}(\sin x) = \pi - x$ and $\cos^{-1}(\cos x) = x \Rightarrow f(x) = (\pi - x) + x = \pi$.

• For $x \in [-\pi, 0]$: f(x) is non-negative (since $\cos^{-1}(\cos x) = -x \geq |\sin^{-1}(\sin x)|$), so range on $[-\pi, 2\pi]$ is $[0, 2\pi]$.

Hence, f is piecewise linear and takes the constant value π on the interval $[\pi/2, \pi]$.

A

[Q.16] Bag X contains 3 red and 4 black balls; Bay Y contains 5 red and 3 black balls. One ball is transferred at random from X to Y, and then one ball is drawn at random from Y. The probability that the drawn ball is red equals:

[A] $\frac{38}{63}$ [B] $\frac{5}{9}$ [C] $\frac{37}{63}$ [D] $\frac{17}{28}$

[ANS] **A**

[SOLN] Bag X has 7 balls (3R, 4B); Bag Y initially has 8 balls (5R, 3B), becoming 9 balls after transfer.

• Case 1: Red ball transferred from X to Y (prob = 3/7). Bag Y then has 6R, 3B ⇒ P(Red from Y) = 6/9.

• Case 2: Black ball transferred from X to Y (prob = 4/7). Bag Y then has 5R, 4B ⇒ P(Red from Y) = 5/9.

Total Probability = (3/7)(6/9) + (4/7)(5/9) = (18 + 20) / 63 = 38/63.

[Q.17] The maximum area of a quadrilateral with sides 1.0 cm, 4.0 cm, 7.0 cm, 8.0 cm is

[A] $\sqrt{799}$ cm^2 [B] 3.5 cm^2 [C] 18.0 cm^2 [D] 11.06 cm^2

[ANS] **C**

[SOLN] By Bretschneider's formula, a quadrilateral with fixed side lengths achieves maximum area when it is cyclic, given by Brahmagupta's formula: Area = $\sqrt{[(s-a)(s-b)(s-c)(s-d)]}$.

Semi-perimeter s = (1 + 4 + 7 + 8) / 2 = 20 / 2 = 10 cm.

Max Area = $\sqrt{[(10-1)(10-4)(10-7)(10-8)]} = \sqrt{(9 \times 6 \times 3 \times 2)} = \sqrt{324} = 18.0\ cm^2$.

[Q.18] A sequence is defined by $a_1 = 1$, $a_{n+1} = a_n + \frac{1}{a_n^2}$. Choose the correct option from the following.

[A] $\lim_{n\to\infty} a_n = 0$ [B] $\lim_{n\to\infty} a_n$ is a finite number

[C] $a_{3000} > 30$ [D] $a_{3000} < 30$

[ANS] **C**

[SOLN] Square both sides of the recurrence:

$$a_{n+1}^2 = \left(a_n + \frac{1}{a_n}\right)^2 = a_n^2 + 2 + \frac{1}{a_n^2} > a_n^2 + 2$$

Summing this inequality from k = 1 to 999:

$a_{1000}^2 > a_1^2 + 2(999) = 1 + 1998 = 1999$.

Since 302 = 900 < 1999 (in fact $\sqrt{1999} \approx 44.71$), we have $a_{1000} > 30$.

[Q.19] Consider the following functional equation for a real valued function on real numbers
$f(x + y) + f(x – y) = 2f(x)f(y)$
and f is not identically zero function.
Choose the correct option from the following :
[A] f is an odd function
[B] f is always differentiable on the set of real numbers
[C] f is always a periodic function
[D] f(0) = 1

[ANS] **D**

[SOLN] This is d'Alembert's functional equation.
Put y = 0: $f(x) + f(x) = 2f(x)f(0) \Rightarrow 2f(x)[1 – f(0)] = 0$.
Since f is not identically zero, there exists x_0 such that $f(x0) \neq 0$, which forces $1 - f(0) = 0 \Rightarrow f(0) = 1$.
(Furthermore, setting x = 0 gives $f(-y) + f(y) = 2f(0)f(y) = 2f(y) \Rightarrow f(-y) = f(y)$, so f is an even function, e.g., f(x) = cos(kx) or cosh(kx), which is not always periodic).

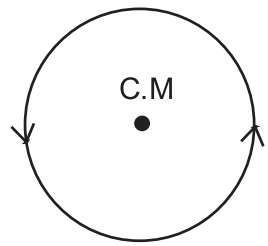
[Q.20] A person claims to have followed this path: She was standing at location 'A' on the Earth. She walked towards the north direction for 100 km. Next, she turned towards east and walked for 100 km. Then she turned towards south and walked for 100 km and reached back the same location 'A'.
[A] her claim is not possible.
[B] Location A can be only one particular place on the Earth.
[C] Location A can be only at two distinct places on the Earth.
[D] Location A can be anywhere on the Earth.

[ANS] **D**

[Q.21] For a binary star system with two identical mass stars,
[A] the orbit is always circular.
[B] the centre of mass of the system shows a periodic motion.
[C] the elliptical orbit may appear to be circular to the observe due to certain inclination
[D] the stars can be always resolved using powerful telescope.

[ANS] **C**

[SOLN] For a binary star system with two identical mass



The elliptical orbital may appear to be circular to the observes due to certain indination.

[Q.22] Two locations on the southern hemisphere of the earth have exactly the same length of day (and night) on any given day, but the time of sunrise and sunset both are offset by twelve hours each. A plane flying between the two locations trying to cover the shortest path will always fly

[A] first southwards and then northwards [B] first northwards and then southwards

[C] exactly westwards. [D] exactly eastwards.

[ANS] **A**

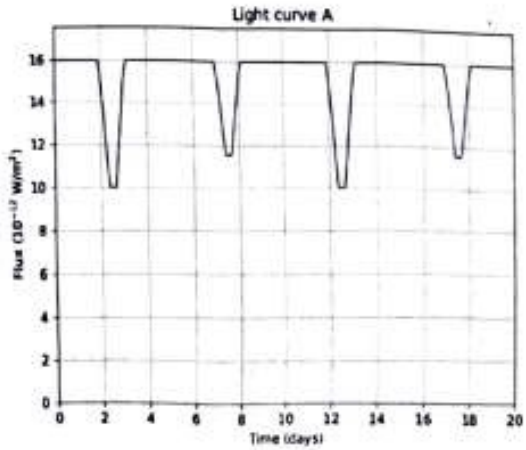
[SOLN] Equal day length implies that the two places have equal absolute latitude.

Sunrise differing by 12 hrs $\rightarrow$ 180° (longitude difference)

For two location in southern hemisphere of opposite longitudes.

Shortest path must be first towards southward and then returns northwards.

[Q.23] The light curve shows a fictional edge-on eclipsing binary system containing star X and star Y. Which of the following is the correct option about the light curve ?



[A] Two stars revolve around common centre of mass is necessarily circular orbit.

[B] Both stars are surely not of the same size.

[C] The two stars have their intensities in the ratio of 6 : 5.

[D] Larger dip is when cooler star comes in front of hotter star.

[ANS] **D**

[SOLN] Flux loss during eclipses = 16 – 10 = 6 = $F_1 \times 10^{-12}$ w / m^2 $= 16 - 11.5 = 4.5 = F_2 \times 10^{-12}$

$$\Rightarrow \frac{F_1}{F_2} = \frac{I_1}{I_2} = \frac{6}{4.5}$$

Larger dip is when cooler star comes in front of hotter star.

[Q.24] An astronomical telescope has objective with focal of length of 750 mm. When eyepiece of 25 mm is used to observe a binary star, it is barely separated to a normal human eye. The aperture of telescope is 150 mm. The actual angular separation of the binary stars is approximately.

[A] 1 arc sec [B] 2 arc sec [C] 1 arc min [D] 2 arc min

[ANS] **B**

[SOLN] $f_o = 750\ mm,\ \ f_e = 25\ mm$

Angular magnification $\frac{f_o}{f_e} = \frac{750}{25} = 30$

$\Rightarrow$ Aperture of telescope is 150 mm.

If the separation is barely separated, then separation = 1 arc min

(resolving power) = 60 arc sec.

Angular separation $= \frac{R.P.}{M} = \frac{60\ arc\ sec.}{30} = 2\ arc\ sec.$

[Q.25] An observer is located at latitude of 60° south and be can only see the sky above 10° horizon due to local geography. What percentage of sky will be never see from his location ?

[A] 17.1% [B] 25.0% [C] 32.9% [D] 41.3%

[ANS] **C**

[SOLN] At latitude of 60° south, star of declination

δ reaches a maximum altitude = $90° - |-60° - \delta| = 30 - \delta < 10°$

$\Rightarrow\ \delta > 20°$

Fraction of full celestial sphere = $\frac{1 - \sin\delta\,min}{2} \approx 0.329\ \%$

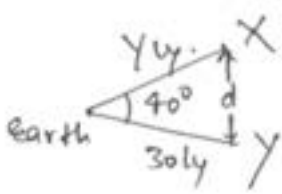
[Q.26] Brightness of stars is given a magnitude. If the flux of energy received from two stars is I_1 and I_2, their magnitudes (m_1 and m_2 respectively) are related a $m_2 - m_1 = -2.5 \log\left(\frac{I_2}{I_1}\right)$. A particular star X is seen with magnitude 3.0 from the earth and with magnitude 4.0 from an exoplanet around another star Y. From the Earth, stars X and Y are seen separated by 40' with each other. The distance of the star Y from the Earth is 30 ly. The distance of the star X from the exoplanet is

[A] 23.6 ly [B] 21.5 ly [C] 13.5 ly [D] 9.4 ly

[ANS] **B**

[SOLN] Brightness of star I_1, I_2

Flux of energy received m_1 & m_2 $\ m_2 - m_1 = -2.5 \log\left(\frac{I_2}{I_2}\right)$



$I_1 = \frac{p}{y_x^2},\ I_2 = \frac{p}{r_y^2} \qquad \Rightarrow \frac{I_2}{I_1} = \left(\frac{r_y}{r_x}\right)^2 \qquad \Rightarrow 4 - 3 = -2.5\log\left(\frac{r_y}{r_x}\right)^2 = -5\log\left(\frac{r_y}{r_x}\right)$

$$I = 5\log\left(\frac{r_x}{r_y}\right) \Rightarrow 10^{\frac{1}{5}} = \frac{I_x}{I_y} \qquad \Rightarrow \quad r_x = r_y \times 10^{\frac{1}{5}} = 1.585\ r_y$$

now, $d^2 = r_x^2 + r_y^2 - 2r \times r_y \cos 40^\circ$

$$d = \sqrt{(1.585\times 30)^2 + (30)^2 - 2\times 1.585\times 30\times 30\times \cos 40^\circ} \qquad d = 21.5 \text{ ly}$$

[Q.27] Which of the following constellations has smallest area?

[A] Triangulum [B] Sagitta [C] Scutum [D] Crux

[ANS] **D**

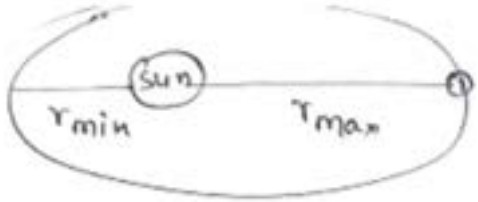
[SOLN] Conceptual.

[Q.28] A comet is orbiting around the sun with the closest distance as 3.00 AU. If its period of revolution around the Sun is 6.55 years, what is its farthest distance from the Sun ?

[A] 3.50 AU [B] 3.00 AU [C] 5.38 AU [D] 8.38 AU

[ANS] **A**

[SOLN]



$$r_{max} = a(1+e) \Rightarrow \frac{r_{max} + r_{min}}{2} = a \qquad r_{min}\, a = (1-e) \quad r_{min} = 3\ AU$$

$$T \propto r^3 \Rightarrow T^2 = \frac{4\pi^2}{GM_s} a^3 \qquad \Rightarrow (6.55)^2 = \frac{4\pi^2 \times a^3}{1.327\times 10^{20}}$$

$$a = \left[\frac{(6.55)^2 \times 1.327\times 10^{20}}{4\pi^2}\right]^{\frac{1}{3}} \approx 3.5\ AU \qquad \text{so, } r_{max} = 2a - r_{min} = 2a - 3\,AU \quad r_{max} = 4\,AU$$

[Q.29] Consider two statements A and R.

Statement A : The ion tail of a comet is always seen on the opposite side of the sun.

Statement R : Ion tail has very low density and hence it lags behind the comet approaching the Sun.

Choose the most appropriate option that best describe the above two statements.

[A] Both statements are correct and statement R is correct reason for statement A.

[B] Both statements are correct, but statement R is correct reason for statement A.

[C] Statement A is correct, but statement R is wrong.

[D] Statement A is wrong, but statement R is correct.

[ANS] **C**

[SOLN] The ion tail of a comet is always seen on the opposite side of the sun.
The ion tail direction is primarily controlled by the Comet's motion.

[Q.30] It is observed that large number of stars follow $L \propto M^4$ relation between luminosity L and mass M. Similarly large number of stars have $R \propto M^{0.8}$. The surface temperature of stars will have relation $T \propto M^x$. The value of x is

[A] 3.2 [B] 1.7 [C] 0.8 [D] 0.6

[ANS] **D**

[SOLN] $L \propto m^4$ & $R \propto M^{0.8}$

Surface temperature $T \propto M^x$ of stars. $\Rightarrow P = 4\pi R^2 = L \Rightarrow T \propto \frac{L^{\frac{1}{4}}}{R^{\frac{1}{2}}}$

$\Rightarrow \frac{T \propto (M^4)^{\frac{1}{4}}}{(M^{0.8})^{\frac{1}{2}}} \Rightarrow T \propto M^{1-0.4} \Rightarrow T \propto M^{0.6}$ x = 0.6

[Q.31] Cosmological redshift

[A] is due to actual motion of galaxies away from the Milky Way.

[B] can be measured due to transverse motion of the sources as seen by the observer.

[C] is due to the cumulative expansion of space along the photon's path from the source to the observer.

[D] is due to slowing down of time near heavy stars and galaxies.

[ANS] **C**

[SOLN] Cosmological redshift is due to the cumulative expansion of space along the photon's path from the source to the observer.

$(\text{Re shift}, \lambda_{obs} > \lambda_{emitted})$

$$\text{redshift} = 1 + Z = \left(\frac{\lambda_{obs}}{\lambda_{emitted}}\right) \Rightarrow \frac{\lambda_{obs}}{\lambda_{amitted}} > 1$$

$\Rightarrow Z = 0$

[Q.32] Planetary scientists are interested in the study of asteroids because asteroids are

[A] numerous and can easily be found anywhere and hence help to study stability of orbits.

[B] chemically undifferentiated due to their smaller mass and hence chemical composition of solar system can be determined.

[C] chemically volatile and reflect lot of light, hence can tell us more about their orbits.

[D] less massive and hence have very stable orbits.

[ANS] **B**

[SOLN] Interest in the study of asteroids is due to Chemically undifferentiated due to their smaller mass and hence chemical composition of solar system can be determined.

[Q.33] In a true scale model of the Solar system, the Earth's size is represented by a one-rupee coin (diameter approximately 2.2 cm). At what average distance can the Jupiter be placed from this one-rupee coin?

[A] 12.2 km [B] 1.22 km [C] 122 m [D] 12.2 m

[ANS] **B**

[SOLN] Earth's diameter = 12,472 km $\equiv$ 2.2 cm

$\Rightarrow$ Scale is reduced by a factor of $\frac{2.2 \text{ cm}}{12.472 \text{ km}} \approx 1.73 \times 10^{-9}$

Avg. distance from Jupiter = $5.2 \text{ AU} = 7.78 \times 10^{8}$

So, avg, distance = $7.78 \times 10^{8} \times 1.73 \times 10^{-9} \times 1 = 1.22 \text{ km}$

[Q.34] Period of satellite Io around the planet Jupiter is 42.5 hours. It is tidally locked to Jupiter. Period of Jupiter around the Sun is 11.86 years while period of its rotation is 9 h 56 m. An observer located on the Equatorial region of Io, is seeing Jupiter almost overhead while the Sun on the Eastern horizon. In how much minimum time will she see the Sun on her Western horizon?

[A] 4.96 hours [B] 21.25 hours

[C] 5.93 years [D] Sun will not change position in the sky

[ANS] **C**

[SOLN] Lo is tidally locked to Jupiter with an orbital period of 42.5 hours, it's rotation period relative to inertial space is also 42.5 hours (solar day is nearly the same).

$\Rightarrow$moving from eastern horizon to the western it takes $\frac{42.5}{2} = 21.25$ hours

& solar day on Lo is

$\frac{1}{T_{solar}} = \frac{1}{T_{Io}} - \frac{1}{T_{Jupiter}}$ & $T_{Jupiter} >> T_{Io} \Rightarrow T_{solar} \approx T_{Io}$

$\Rightarrow \min^{m}$ time to see the sun on the western horizon = half of Jupiter's orbital motion = $\frac{11.86}{2} =$ 5.93 years

[Q.35] Brightness of celestial object is given in magnitude. If the flux of energy received from two objects are I_1 and I_2, then their magnitudes (m_1 and m_2 respectively) are related to fluxes as $m_2 - m_1 = -2.5 \log\left(\frac{I_2}{I_1}\right)$. A space probe from a distance of 50 pc is observing transit of Jupiter in front of the Sun. How much will be the change in magnitude when Jupiter's disc is fully in front of the Sun? Assume Sun's disc is uniformly bright. Sun's diameter is 109 times diameter of the Earth and Jupiter's diameter is 11 times that of the Earth.

[A] 0.01 [B] 0.02 [C] 0.04 [D] 0.5

[ANS] **A**

[SOLN] $\Rightarrow \left(\frac{\Delta F}{F}\right) = \left(\frac{11}{109}\right)^2 = 0.01018$

$\Rightarrow \frac{I_2}{I_1} = 1 - \frac{\Delta F}{F}$

$\Delta m = -2.5\log_{10}(1 - 0.01018)$

$= -2.5\log_{10}\ 0.98982 = 0.011$

[Q.36] A student applies various voltages (V) across a resistor and measures corresponding currents (I). She then plots the graph of I (1cm as 1 mA on Y axis) and V (1 cm as 1 V on X axis). The best fit line makes an angle of $70^0 \pm 1^0$ with voltage axis. The value of resistor can be determined as

[A] $344 \pm 5\Omega$ [B] $344 \pm 20\Omega$ [C] $2.90 \pm 0.04k\Omega$ [D] $2.90 \pm 0.16k\Omega$

[ANS] **B**

[SOLN] $\tan 70^o = \frac{I}{V} \Rightarrow R = 344\Omega$

$\Rightarrow R = \cot\theta\left[\theta = 70\right]$

$\Rightarrow \log_e R = \log_e \cot\theta$

$\Rightarrow \frac{1}{R}\cdot\Delta R = \frac{1}{\cot\theta}\times \operatorname{cosec}^2\theta \times \Delta\theta$

$\Rightarrow \frac{\Delta R}{R} = \frac{\Delta\theta}{\sin\theta\cdot\cos\theta}$

$\Rightarrow \Delta R \approx 344 \times \frac{(\pi/180)}{\sin 70^o \cos 70} \approx 344 \times 0.052 \approx 20\Omega$

$\therefore R = (344 \pm 20)\Omega$

[Q.37] Weight of a cubical block is measured to be 360 gram force with spiring balance having least count of 20 gram force. Side length of the block, measured with least count of 1 mm, is equal to 13.6 cm. Calculation of pressure developed by the block on a table would have the uncertainty of

[A] 3% [B] 5%

[C] 7% [D] will depend on size of the table

[ANS] **C**

[SOLN] $P = \text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{W}{l^2}$

$\Rightarrow \frac{\Delta P}{P}\times 100\% = \left(\frac{\Delta W}{W}\times 100\%\right) + 2\left(\frac{\Delta l}{l}\times 100\%\right)$

$= \left(\frac{20}{360} + \frac{0.2}{13.6}\right)\times 100\%$

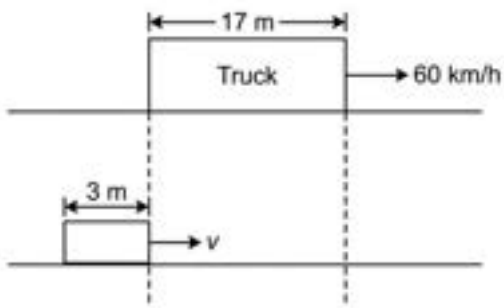
$\approx 7\%$

[Q.38] A tuck of length 17 m is running on a straight highway with uniform speed of 60.0 km-hr. A car of length 3 m is running with zero acceleration on a truck parallel to that of the truck, The car takes 3.6 seconds to completely overtake the truck. The speed of the car must be

[A] 20 km/hr [B] 71 km/hr [C] 80 km/hr [D] 82 km/hr

[ANS] **C**

[SOLN]



In truck frame $\Rightarrow (v-60)=\frac{20}{3.6}\times\frac{18}{5}$

$\Rightarrow v=60=20$

$\Rightarrow v=80\ km/h$

[Q.39] A circular loop of radius R has N closely wound turns of a wire carrying current I. It is suspended vertically in a region of horizontal magnetic field of strength B in such a way that its area vector makes an angle of $30°$ with the magnetic field. Keeping the perimeter of the loop same if it is made into square shape,

[A] If the angle between area vector and B is increased to $60°$ without changing N, I and B the amount of torque will decrease.

[B] With the same N and I, to get the same amount of torque, angle between area vector and B should be increased without changing the magnitude of B.

[C] With the same N, I and orientation of the loop, torque developed will increase if magnitude of B is decreased.

[D] With the same N and orientation of the loop, to get the same torque developed, current I and magnitude of B should be decreased.

[ANS] **B**

[SOLN] $\text{Torque}, \tau = M\times B \Rightarrow \tau = MB\sin\theta$

$= NiAB\sin\theta$

$\text{option1}: \text{As}\sin\theta\uparrow\ \tau\uparrow\rightarrow \text{incorrect}$

$\text{option 2}: \tau_1=\tau_2$

$NI\left(\frac{\pi^2R^2}{4}\right)\times B\times\sin\theta_1 = NI\left(\pi R^2\right)B\sin 30^o$

$\Rightarrow \sin\theta_1=\frac{2}{\pi}\approx 0.6366$

$\Rightarrow \theta > 30^{o} \rightarrow$ Angle should be increased $\rightarrow$ Correct

option 3 $\rightarrow$ incorrect

option 4 $\rightarrow$ incorrect

[Q.40] When a person of average height stands on a smooth surface with clear horizons, the approximate distance to the horizon from the person is

[A] 500 km [B] 50 km [C] 15 km [D] 5 km

[ANS] **D**

[SOLN] Distance to horizon on (spherical)

$\text{earth} \approx \sqrt{2Rh}$

$R \approx 6.4 \times 10^{6}\,\text{m}$

$h \approx 1.8\ \text{m}$

$\Rightarrow d \approx 5\ \text{km}$

[Q.41] Consider following statements:

Statement I: During heavy winters ponds freeze from top.

Statement II: Volume of water increases on heating as well as cooling starting at 4 °C.

Which of the following is correct conclusion.

[A] Both the statements are correct and statement II is the correct reason for statement I.

[B] Both the statements are correct but statement II is not the correct reason for statement I.

[C] Only statement I is correct.

[D] Only statement II is correct.

[ANS] **A**

[SOLN] $\rightarrow$ Pond freezes from top because water density is maximum at 4°C.

$\rightarrow$ Anomalous behaviour at 4°C is reason.

[Q.42] A moving coil galvanometer with a coil having resistance R_G, is being used as a current meter, connecting a shunt resistance $R_s << R_G$ parallel to the coil. The galvanometer now works with current sensitivity of 1 rad/microampere. Magnetic field of the MCG is now doubled and a resistance of 100 kΩ >> R_G is connected in series with its coil so that it can be used as a voltmeter. Voltage sensitivity of this voltmeter would be,

[A] 20 rad/mV [B] 2.0 rad/mV [C] 0.20 rad/mV [D] 0.020 rad/mV

[ANS] **D**

[SOLN] $\theta = \left(\frac{NBA}{K}\right) I$

Sensitivity, $i_s = \frac{\theta}{i} = 1$ rad/μA (initial)

As $i_s \propto B \Rightarrow$ final i_s = 2 x initial sensitivity = 2 rad/μA

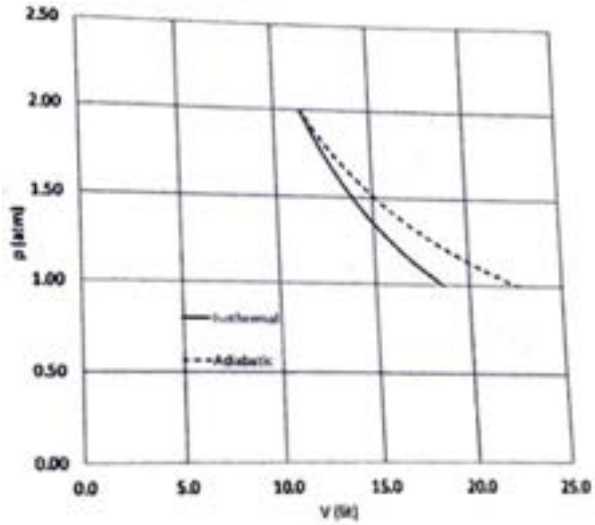
For voltmeter

$$R = R_g + R_m \simeq R_m \simeq 100\,K\Omega$$

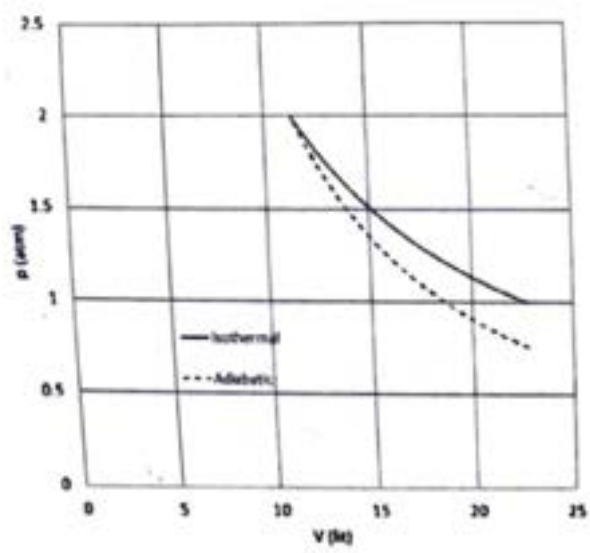
Voltage Sensitivity = $\frac{i_s}{R} = \frac{2\ rad/\mu A}{100\ K\Omega}$ = 0.020 rad/mV

[Q.43] A reversible process is an infinitesimally slow process that can be reversed without leaving any change in the system. One mole of air at STP is compressed isothermally and reversibly to one-half of the original volume. It is then expanded adiabatically and reversibly to its original pressure. Which of the following graphs correctly shows these processes?

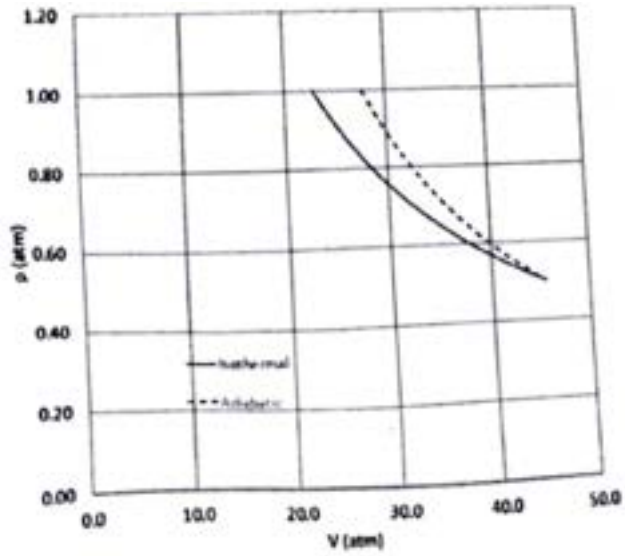
[A]



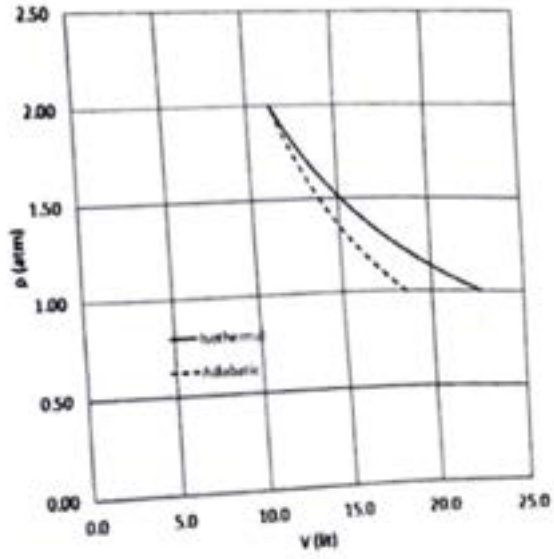
[B]



[C]



[D]



[ANS] D

[SOLN] One mole of air at STP:

$P_1 = 1$ atm

$V_1 \approx 22.4$ L

$T_1 = 273.15$ K

Solid line → Isothermal

So, $P_1V_1 = P_2V_2$ $(V_2 = V_1/2)$

$P_2 = 2$ atm

So, option (c) → incorrect

Dashed line → Adiabatically; $P_3 \approx P_1$ (given)

So, $P_2V_2^{\gamma} = P_3V_3^{\gamma}$

$V_3 = V_2 \times (P_2/P_1)^{1/\gamma}$

$= 11.2 \times 2^{1/1.4} \approx 18.4$ L

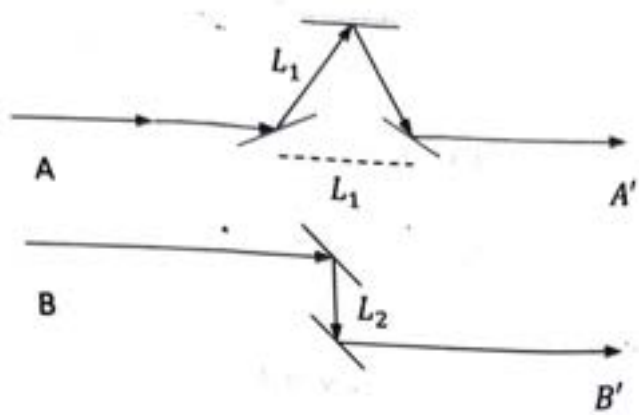
So, option (b) → incorrect.

And slope of adiabatic > slope of isothermal.

So, option (a) → incorrect

Option (d) → Correct.

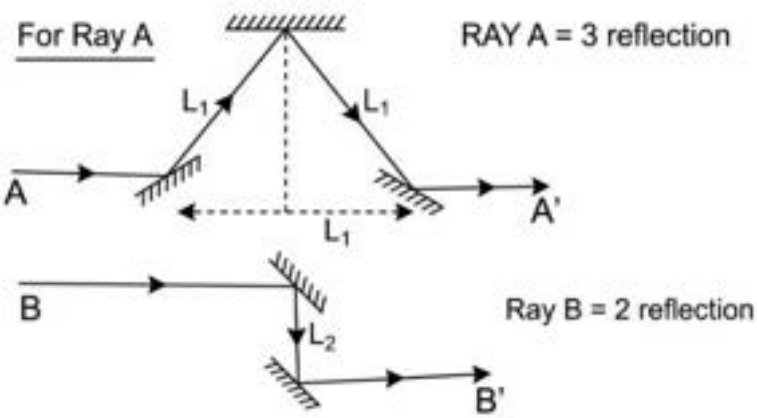
[Q.44] Two rays A & B are light waves of equal wavelength λ and they are in phase. Ray A gets reflected at three mirrors and ray B at two mirrors as shown in the figure. Distance between consecutive reflecting faces for and that for A is L_1 and that for B is L_2 such that $L_2 = L_1/3$. The smallest value of L_1 giving A' and B' in phase is



[A] $\frac{3\lambda}{4}$ [B] $\frac{3\lambda}{5}$ [C] $\frac{3\lambda}{10}$ [D] $\frac{3\lambda}{15}$

[ANS] **A**

[SOLN]



$\Delta x_A = 2L_1 + 3\lambda/2$ (for each reflection)

$\Delta x_B = (L_1/2 + L_2 + L_1/2 + 2\lambda/2)$

$\Delta x = \Delta x_A - \Delta x_B$

$= 2L_1 + 3\lambda/2 - (4 + 4/3 + \lambda)$

$= 2L_1 - 4L_1/3 + \lambda/2$

$= 2L_1/3 + \lambda/2$

For same phase

$2L_1/3 + \lambda/2 = n\lambda$, where {n = 1, 2, …

$L_1 = 3\lambda/4$

[Q.45] Imagine a new scale for measuring sound energy is designed. In this scale, I μJ energy incident per unit per square meter, is denoted as 0 sonus. Every increase of 4 sonus increases the sound energy by 100 times. Two identical sound sources of rating 3.2 sonus are played simultaneously. It was observed that one of them has its frequency slightly shifted, so they create beats. What will be the highest level of beats?

[A] 3.2 sonus [B] 3.8 sonus [C] 4.4 sonus [D] 6.4 sonus

[ANS] **C**

[SOLN] $I_0 = 1\ \mu W / m^2$ *(Given)*

Now,

$I / I_0 = (100)^{L/4}$ *{ L = Sound level in sonus }*

$I / I_0 = (10^2)^{L/4}$

$I / I_0 = 10^{L/2}$

Taking log on both sides:

$L = 2 \log(I / I_0)$

Now,

$3.2 = 2 \log(I_1 / I_0)$

$\log(I_1 / I_0) = 1.6$

$I_1 = I_0 \times 10^{1.6}$ **— (1)**

Max Intensity during Beats

$A_{max} = A_1 + A_2$ *{ two identical sound waves }*

$A_{max} = 2A_1$ $I_{max} = (2)^2 I_1$

$I_{max} \propto A^2$ $I_{max} = 4 \times I_0 \times 10^{1.6}$ **— (2**

$$\therefore I_{max} = 2 \log_{10}\left(\frac{4 \times I_0 \times 10^{1.6}}{I_0}\right)$$

$= 2\log_{10}\left(4 \text{ x } 10^{1.6}\right)$

$L_{max} = 2\left[\log 4 + \log 10^{1.6}\right]$

$= 2\left[0.602 + 1.6\right]$

$= 2 \text{ x } 2.202$

$= 4.4 \text{ sonus}$

[Q.46] A radioactive sample of Na-24 used for irradiation of a patient is prepared exactly 6.0 hrs before it is to be used for a patient. It has half-life of 14.96 hrs. If it is expected to have activity of 40 kBq at the start of the treatment, what should be its mass when it is prepared?

[A] 5.90 X 10^{-4} μg [B] 2.46 x 10^{-5} μg [C] 3.94 x 10^{-6} μg [D] 1.64 x 10^{-7} μg

[ANS] **D**

[SOLN] M = 24 g/mol

$t_{1/2}$= 14.96 hrs

= 14.96 × 3600

= 53,856 second

t = 6 hrs

A = 40 kBq = 40 × 10^3 Bq

$\Rightarrow \quad A = A_0\left(1/2\right)^{\frac{t}{t_{1/2}}}$

$A_0 = A.2^{\frac{t}{t_{1/2}}}$

$A_0 40 \text{ x } 10^3 \text{ x } 2^{\frac{6}{14.96}}$

≈ 5.282 × 10^4 Bq

For Activity:

$A_0 = \lambda N_0$

$\lambda = \ln \frac{2}{t_{1/2}} = 0.693 / 53856 = 1.287 \times 10^{-5}\ s^{-1}$

$N_0 = A_0 / \lambda = (5.282 \times 10^4) / (1.287 \times 10^{-5}) = 4.10 \times 10^9$

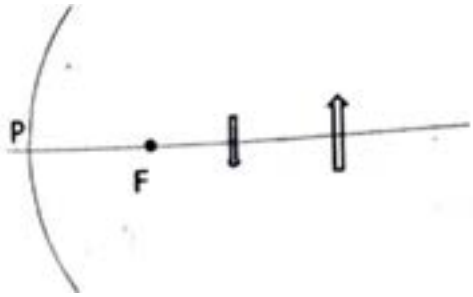
Now:

$m = (N_0 / N_A) \times M = (4.10 \times 10^9 / 6.022 \times 10^{23}) \times 24 \approx 1.636 \times 10^{-13}$ g

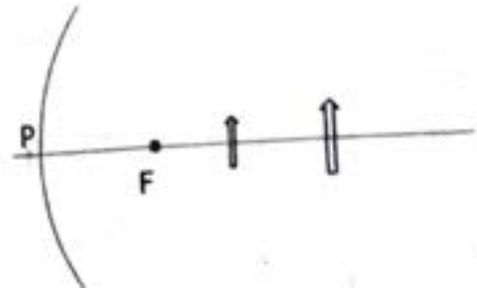
$= 1.64 \times 10^{-7}$ μg

[Q.47] A caterpillar is in front of a concave mirror of focal length f as shown in a diagram. It is located at a distance of 2.5 f from the pole of the mirror and its height, perpendicular to the axis is 1.2 cm. Select the correct image of caterpillar for this situation. Caterpillar is shown with an upright arrow at the same location in all images. The other arrow is the image.

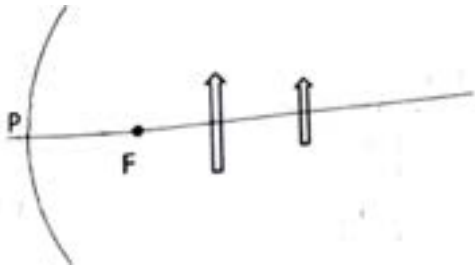
[A]



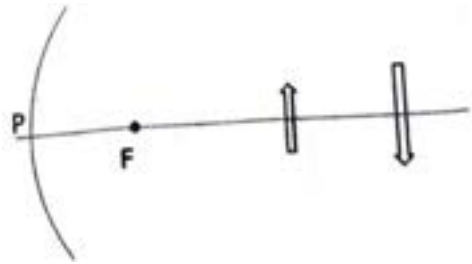
[B]



[C]



[D]



[ANS] **A**

[SOLN] Given:

$$u = -2.5f$$

$$h_0 = 1.2\ \text{cm}$$

Step 1: Find Image Distance (v) using the Mirror Formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{-f} = \frac{1}{v} - \frac{1}{2.5f}$$

$$\frac{1}{v} = -\frac{1}{f} + \frac{1}{2.5f}$$

$$\frac{1}{v} = -\frac{1}{f} + \frac{0.4}{f} = -\frac{0.6}{f}$$

$$v = -\frac{f}{0.6} = -1.67f = -\frac{5}{3}f$$

Step 2: Find Magnification (m) and Image Height (h_i)

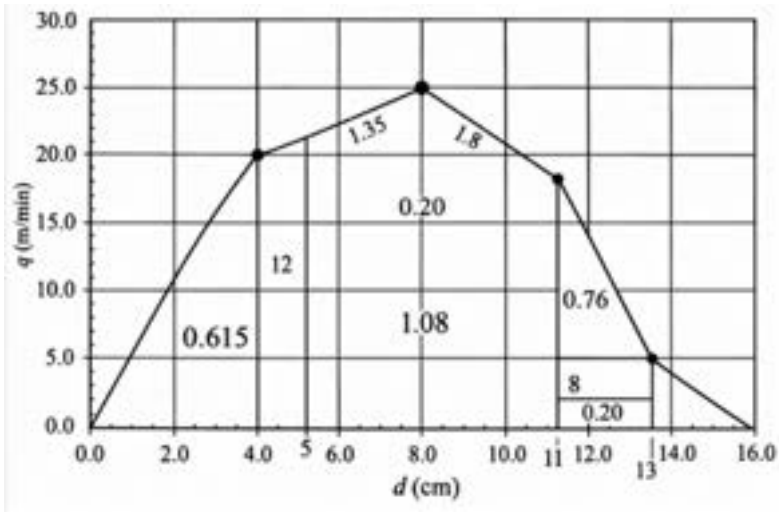
$$m = \frac{h_i}{h_o} = -\frac{v}{u}$$

$$m - \frac{-\frac{5}{3}f}{-\frac{5}{2}f} = -\frac{2}{3}$$

$$h_i = m \times h_o = -\frac{2}{3} x 1.2 = -0.8\ \text{cm}$$

Nature of Image: Real and inverted

[Q.48] A proton moves in variable electric field as shown in the graph. If original speed of proton is 2.00 km/s, what is its final speed?



[A] 22.4 km/s [B] 20.5 km/s [C] 205 km/s [D] 880 km/s

[ANS] **B**

(NSEA) PART : A–2

ANY NUMBER OF OPTIONS (4, 3, 2 or 1) MAY BE CORRECT

MARKS WILL BE AWARDED ONLY IF ALL CORRECT OPTIONS ARE BUBBLED AND NO INCORRECT.

[Q.49] Let $a_n = x^n + \frac{1}{x^n}$, where $x + \frac{1}{x} = 3$. Which of the following are correct?

[A] $a_n = 3a_{n-1} - a_{n-2}$ [B] $a_2 = 7$ [C] $a_4 = 49$ [D] $a_6 = 322$

[ANS] **A,B,D**

[SOLN] Given $a_n = x^n + x^{-n}$ and $x + 1/x = 3$.

Multiplying the given relation by x^{n-1} and using the same relation, we obtain the recurrence:

$a_n = x^n + x^{-n}$

$3a_{n-1} = (x + 1/x)(x^{n-1} + x^{-(n-1)})$

$= x^n + x^{n-2} + x^{-n+2} + x^{-n}$

$= a_n + a_{n-2}$.

Hence, $a_n = 3a_{n-1} - a_{n-2}$. Also, $a_0 = 2$ and $a_1 = 3$.

$a_2 = 3(3) - 2 = 7$; $a_3 = 3(7) - 3 = 18$; $a_4 = 3(18) - 7 = 47$.

$a_5 = 3(47) - 18 = 123$; $a_6 = 3(123) - 47 = 322$

[Q.50] Let $\Delta = \begin{vmatrix} a+b & c & c \\ c+a & b & a \\ b+c & a & b \end{vmatrix}$ .Which of the following are correct ?

[A] $\Delta = 0$ [B] $a+b+c=0 \Rightarrow \Delta = 0$

[C] Δ changes sign, if a and b are interchanged [D] Δ is divisible by (a – b)

[ANS] **B,C,D**

[SOLN] Given $\Delta = |\,a+b \quad c \quad c\,;\quad c+a \quad b \quad a\,;\quad b+c \quad a \quad b\,|$.

Expanding and factorising the determinant gives

$\Delta = -(a - b)(a + b - 2c)(a + b + c)$.

(a) False in general: the determinant is not identically zero.

(b) True: if $a + b + c = 0$, then $\Delta = 0$.

(c) True: interchanging a and b changes the sign of the factor (a - b), while the other factors remain unchanged; hence Δ changes sign.

(d) True: the factorisation contains (a - b), so Δ is divisible by (a - b).

[Q.51] Let $\vec{a}, \vec{b}$ and $\vec{c}$ be the three non-coplanar vectors to define other three vectors $\vec{p}$, $\vec{q}$ and $\vec{r}$ such that $\vec{p} = \dfrac{\vec{b}\times\vec{c}}{[\vec{a}\,\vec{b}\,\vec{c}]}$, $\vec{q} = \dfrac{\vec{c}\times\vec{a}}{[\vec{a}\,\vec{b}\,\vec{c}]}$ and $\vec{r} = \dfrac{\vec{a}\times\vec{b}}{[\vec{a}\,\vec{b}\,\vec{c}]}$.

Which of the following is/are necessarily frue?

[A] $\vec{a}\cdot\vec{p}+\vec{b}\cdot\vec{q}+\vec{c}\cdot\vec{r} = 3$

[B] $\vec{a}\cdot\vec{q} = 0$

[C] $[\vec{p}\,\vec{q}\,\vec{r}] = \dfrac{1}{[\vec{a}\,\vec{b}\,\vec{c}]}$

[D] $\vec{p}, \vec{q}, \vec{r}$ are coplanar

[ANS] **A,B,C**

[SOLN] Let D = [a b c] = a · (b × c). Since a, b and c are non-coplanar, $D \neq 0$. The vectors are defined by p = (b × c)/D, q = (c × a)/D and r = (a × b)/D.

Using the scalar triple product:

a · p = [a b c]/D = 1, b · q = [b c a]/D = 1, c · r = [c a b]/D = 1.

Therefore, a · p + b · q + c · r = 1 + 1 + 1 = 3. So (a) is true.

Also, a · q = a · (c × a)/D = 0, because the scalar triple product has two equal vectors. Thus (b) is true.

The vectors p, q and r form the reciprocal basis. Their scalar triple product is [p q r] = 1/D = 1/[a b c] $\neq$ 0. Hence (c) is true. A non-zero scalar triple product means p, q and r are non-coplanar, so (d) is false.

[Q.52] An ellipse has foci at F_1 (1, 0) and h (–1, 0) and passes through the point $P\left(1, \dfrac{1}{\sqrt{2}}\right)$. Which of the following are correct?

[A] $a^2 = 2, b^2 = 1$

[B] The eccentricity is $e = \dfrac{1}{\sqrt{2}}$.

[C] The length of the minor axis is 2.

[D] The directrices of the ellipse are x = ±2.

[ANS] **A,B,C,D**

[SOLN] The foci are F_1 (1, 0) and F_2 (-1, 0). Thus the major axis is along the x-axis and c = 1. The ellipse has equation $x^2/a^2 + y^2/b^2 = 1$, with $a^2 = b^2 + 1$.

Since P(1, 1/ 2) lies on the ellipse:

$1/a^2 + (1/2)/b^2 = 1$.

Put $b^2 = B$, so $a^2 = B + 1$. Then

$1/(B + 1) + 1/(2B) = 1$

- $2B + (B + 1) = 2B(B + 1)$
- $2B^2 - B - 1 = 0$
- $(2B + 1)(B - 1) = 0$.

Since b^2 must be positive, B = 1. Therefore $b^2 = 1$ and $a^2 = 2$.

(a) True: $a^2 = 2$ and $b^2 = 1$.

(b) True: e = c/a = 1/ 2.

(c) True: minor-axis length = 2b = 2.

(d) True: directrices are $x = \pm a/e = \pm 2/(1/ 2) = \pm 2$

[Q.53] Inside the core of star,

[A] the p-p chain of nuclear fusion creates He from H in a solar mass or a less massive star.

[B] the CNO cycle creates the nuclei of elements N and O.

[C] all heavy nuclei till Fe are created if star is massive and evolved.

[D] the triple alpha process creates the nuclei of C.

[ANS] **A,C,D**

[SOLN] (A) Proton-proton pair $_1H^1 + {}_1H^1 + {}_1H^1 + {}_1H^1 \rightarrow {}_2^4He + 2e^+ + 2\nu e + \theta$

(C) multiple chain of nuclear fission

(D) $3\,{}_2He^4 \rightarrow {}_6C^{12}$

[Q.54] A hypothetical planet-star system, behaves exactly like the Earth-Sun system (rotation and revolution period of the planet are the same as that of Earth), with only one difference – the axis of the rotation of the planet is in the orbital plane.

[A] The star will appear always on the zenith to the observer at north pole.

[B] When the star sets (or rises) for the observer at the poles, the star will be at the zenith at mid-day for the observer at the equator.

[C] The star will always appear at the horizon to the observer at the equator.

[D] An observer on either of the poles of the planet will see the sun rise once and set once in one year cycle.

[ANS] **B,D**

[SOLN] Conceptual

[Q.55] For an observer located at the equator of the Earth, moving in an elliptical orbit around the Sun,

[A] duration between two sunrises is always the same.

[B] the sun passes over zenith every 24 hours.

[C] a star can rise twice on a particular date.

[D] if two stars rise together, they also set together.

[ANS] **D**

[SOLN] Diurnal rotation is a rigid rotation of the celestial sphere, so two stars rising together also set together.

[Q.56] Two planets, A and B, move around the same star with exactly the same time period. Planet A and planet B will

[A] always be exactly on opposite sides of the star.

[B] sweep out exactly equal areas in equal intervals of time.

[C] always have the same distance of closest approach to the star.

[D] have the same distance of farthest approach from the star.

[ANS] **B**

[SOLN] $T^2 \propto a^3 \quad \Rightarrow \quad \frac{dA}{dt} = \frac{\pi ab}{T} = \frac{\pi \cdot a \cdot b}{ca^3}$

$T_A = T_B \Rightarrow a_A = a_B \quad \& \quad r_{max} = a(1+e) \quad \& \quad r_{min} = a(1-e)$

$\Rightarrow \quad b^2 = a(1-e^2)$ So, $\frac{dA}{dt}$ depends on 'a'.

[Q.57] A current carrying conductor is kept horizontal along North – South. It carries current from South to North. A small magnetic needle, free to rotate about vertical axis can be held in the neighbourhood of the conductor. If the needle is held

[A] above the conductor, its North pole deflects towards East.

[B] below the conductor, its South pole deflects towards East.

[C] East of the conductor in the same horizontal plane, North pole deflects towards East.

[D] West of the conductor in the same horizontal plane, South pole deflects towards East.

[ANS] **A, B**

[Q.58] A charge given to a solid copper sphere, kept on insulator stand, distributes uniformly on its surface only. Which of the following principles is/are involved in this fact ?

[A] Like charges repel each other.

[B] Copper is a good conductor.

[C] Shape of the conductor is sphere.

[D] Every system tries to minimize its potential energy.

[ANS] **A,B,C,D**

[Q.59] Assume Earth as a sphere with density radially increasing from surface (3 g/cc) to the core (>15 g/cc). Which of the following statements is/are correct statements ?

[A] Gravitational field at height h above the surface will never be greater than field on the surface.

[B] Gravitational field at some depth d below the surface can be greater than field on the surface.

[C] Gravitational potential will always decrease as one moves towards the centre from surface.

[D] Gravitational potential near the centre of the Earth can be taken zero for calculations.

[ANS] **A,B,C,D**

[Q.60] If the Planck's constant increases by 1 order of magnitude and speed of light reduces by one order of magnitude, without changing any other constants, which of the following will hold true?

[A] Energy of hydrogen atoms will reduce by 2 orders of magnitude

[B] Wavelength of hydrogen spectral lines will increase by 2 orders of magnitude

[C] Electron's rest mass energy will reduce by 2 orders of magnitude

[D] Moment of inertia of diatomic molecules will increase by 2 orders of magnitude

[ANS] **A,B,C**